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AN ALL-SPEED RELAXATION SCHEME FOR THE SIMULATION OF MULTI-MATERIAL FLOWS

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ABSTRACT

We propose an implicit all-speed scheme for the simulation of compressible flows inside gases, liquids and solids in different regimes. The scheme accurately simulates the propagation of material waves. A numerical model for the description of physical interfaces will also be introduced.

INTRODUCTION

We introduce an all-speed relaxation scheme for the simulation of compressible material flows at all speeds. A monolithic Eulerian model describing gases, fluids and elastic solids with the same system of conservation laws is adopted [1].

The numerical scheme is based on the relaxation technique introduced by Jin and Xin [2]. We adopt a fully implicit time integration thanks to the linearity of the transport operator in the relaxation system. The spatial discretization is obtained by a combination of upwind and centered schemes, in order to recover the correct numerical viscosity at all Mach numbers [1].

EULERIAN MODEL FOR COMPRESSIBLE MATERIALS

The conservative form of the equations of a general medium in the deformed configuration is given by

$$\begin{cases} \partial_t \rho + \mathrm{div}_x(\rho u) = 0 \\ \partial_t(\rho u) + \mathrm{div}_x(\rho u \otimes u - \sigma) = 0 \\ \partial_t([\nabla_x Y]) + \nabla_x(u \cdot [\nabla_x Y]) = 0 \\ \partial_t(\rho e) + \mathrm{div}_x\left(\rho e u - \sigma^T u\right) = 0. \end{cases} \tag{1}$$

Here ρ is the density, u is the Eulerian velocity field, σ is the Cauchy stress tensor and $[\nabla_x Y]$ is the gradient of the backward characteristics, describing the solid deformation. e is the total energy per unit mass and it is given by the sum of the kinetic energy and the internal energy per unit mass ε. We adopt the following general constitutive law, which is able to describe gases, fluids and elastic solids at the same time:

$$\varepsilon(\rho, s, [\nabla_x Y]) = \underbrace{\frac{\kappa(s)}{\gamma - 1}\left(\frac{1}{\rho} - b\right)^{1-\gamma} - a\rho + \frac{p_\infty}{\rho}}_{\text{general gas}} + \underbrace{\frac{\chi}{\rho}\left(\mathrm{tr}\overline{B} - 2\right)}_{\text{neohookean solid}}. \tag{2}$$

Here $\kappa(s) = exp(s/c_v)$ (s being the entropy), $\overline{B}$ is the 2D right Cauchy-Green tensor and χ, p_∞, γ, a, b are positive constants characterizing a given material.

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ALL-SPEED RELAXATION SCHEME

Letting $\psi = [\rho, \rho u_1, \rho u_2, Y^1_{,1}, Y^2_{,1}, Y^1_{,2}, Y^2_{,2}, \rho e]$ be the conservative variables and $\mathbf{F}(\psi)$ and $\mathbf{G}(\psi)$ the fluxes in the x_1 and x_2 directions, we adopt the relaxation method introduced by Jin and Xin [2]. The relaxation rate η and the relaxation matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are introduced. The fluxes $\mathbf{F}(\psi)$ and $\mathbf{G}(\psi)$ are "relaxed" by the vectors of relaxation variables $\mathbf{v}$ and $\mathbf{w}$ respectively: thanks to this procedure, the advective operator becomes linear. In this framework, a fully implicit time discretization is easily implemented. By employing finite volumes on a Cartesian mesh, the full scheme at first order reads:

$$\begin{cases} \dfrac{\psi^{n+1}_{ij} - \psi^n_{ij}}{\Delta t} + \dfrac{\mathbf{v}^{n+1}_{i+1/2,j} - \mathbf{v}^{n+1}_{i-1/2,j}}{\Delta x_1} + \dfrac{\mathbf{w}^{n+1}_{i,j+1/2} - \mathbf{w}^{n+1}_{i,j-1/2}}{\Delta x_2} = 0 \\ \dfrac{\mathbf{v}^{n+1}_{ij} - \mathbf{v}^n_{ij}}{\Delta t} + \mathbf{A}_1 \dfrac{\psi^{n+1}_{i+1/2,j} - \psi^{n+1}_{i-1/2,j}}{\Delta x_1} = \dfrac{1}{\eta}\left(\mathbf{F}\left(\psi^{n+1}_{ij}\right) - \mathbf{v}^{n+1}_{ij}\right) \\ \dfrac{\mathbf{w}^{n+1}_{ij} - \mathbf{w}^n_{ij}}{\Delta t} + \mathbf{A}_2 \dfrac{\psi^{n+1}_{i,j+1/2} - \psi^{n+1}_{i,j-1/2}}{\Delta x_2} = \dfrac{1}{\eta}\left(\mathbf{G}\left(\psi^{n+1}_{ij}\right) - \mathbf{w}^{n+1}_{ij}\right). \end{cases} \tag{3}$$

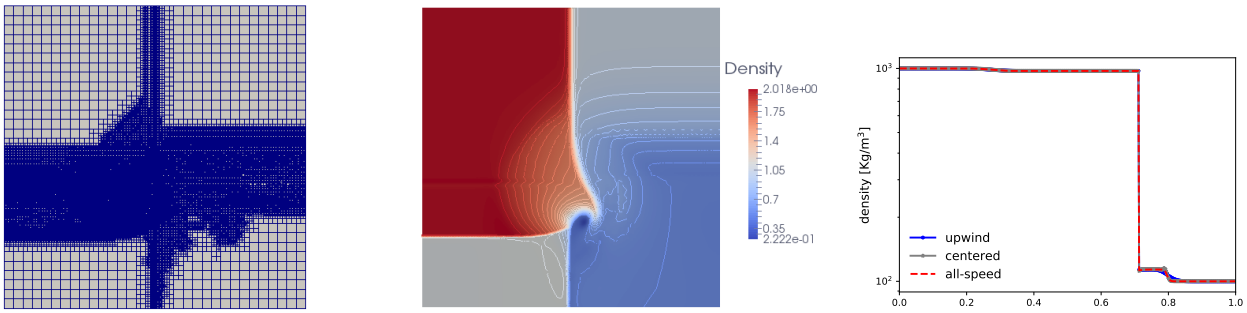
The variables at the interfaces $x_{i+1/2,j}$ inside (3) are computed as follows

$$\mathbf{v}_{i+1/2,j} = f(M_{loc})\left(\mathbf{v}_{i+1/2,j}\right)_{upw} + (1 - f(M_{loc}))\left(\mathbf{v}_{i+1/2,j}\right)_{cent}. \tag{4}$$

This is a convex combination of upwind and centered fluxes, which provides the correct numerical viscosity for each regime, according to the local Mach number M_{loc}. We choose $f(M_{loc}) = \min\{1, M_{loc}\}$.

NUMERICAL VALIDATIONS

As a sample of the numerical results, here we show the results on a 2D Riemann problem (2DRP) involving two contact waves, a shock and a rarefaction in a gas. This is solved with an adaptive mesh refinement technique based on the numerical entropy production criterion [3]. The grid and the density profile obtained at time $t = 0.3$ are shown in Figs. 1(a) and 1(b). The all-speed scheme keeps the contact waves sharp, thanks to (4).



(a) 2DRP: grid (b) 2DRP: density (c) Water/gas interface

FIGURE 1: (a)-(b): 2D Riemann problem in a gas. (c): multi-material flow (water/gas interface).

In Fig. 1(c) we present a preliminary 1D multi-material test, where a water/gas interface is solved. The Mach number in water (left) is around $3 \cdot 10^{-2}$, whereas in the gas (right) is around 0.15, hence we are dealing with a multi-regime test. The transmission conditions are imposed via immersed boundary extrapolations, thus the interface is sharp by construction.

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